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A review on trend factor: examination of the trend factor's performance with skipping period

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Abstract of master's thesis

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Abstract

Han, Zhou and Zhu (2016) proposed a trend factor to capture all the short-, mid- and long-term information which represents the well-known short-term reversal factor, the momentum factor and the long-term reversal factor. HZZ documented the superiority performance of the trend factor with its high and consistent abnormal return.

Based on HZZ's approach, this study provides some further examinations on the trend factor's performance with the skipping period. The skipping period is widely used by related studies in order to mitigate bid-ask spread bias and avoid the opposite effects from shorter-term factors. The skipping period also provides a practical setup which considers the real-life trades execution issues. The study finds that with the skipping period, the performance of the trend factor largely declines and its superiority over other factors disappears. The trend factor's monthly average return drops from 1.69% by more than 0.50% when the 1-day skipping periods are applied, and after applying the 5-day and 20-day skipping periods the return becomes lower than that of the short-term reversal factor and the momentum factor.

The study also shows such impacts of skipping period over the trend factor is mainly due to the short-term reversal factor, and especially the 5-day lag of the trend factor, which accounts for 0.82% out of 1.69% of the trend factor's return.

Keywords factors, trend factor, short-term reversal, momentum, long-term reversal, skipping period, moving average

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1. Introduction

Several types of stock return anomalies, which seem to be in conflicts with the efficient market hypothesis, have been documented by empirical studies, and the abnormal returns claimed by those studies are at the center of research and discussions over the years. Three of the most often discussed anomalies related to the weak form of efficient market hypothesis (or technical analysis) are especially interesting and draw lots of attentions: the short-term reversal effect, documented by Lehmann (1990) and Jegadeesh (1990), describes the phenomenon that the short-term (from few days up to one month) stock returns tend to reverse in the next period; the momentum effect, documented by Jegadeesh and Titman (1993), suggests the tendency of stock performance over a mid-term period (from three to twelve months) is likely to continue; and the long-term reversal effect, documented by De Bondt and Thaler (1985), indicates the reversal effect also exist in the longer-term periods (from one year to few years). Many studies on those three types of anomalies report significant and consistent returns over the years, while the discussions on possible explanations for such abnormal returns are still ongoing.

In the paper "A trend factor: Any economic gains from using information over investment horizons?" Han, Zhou and Zhu (2016) proposed a trend factor, which captures all the short-, mid-, and long-term stock price signals, and generates decent abnormal returns. According to HZZ's results, the trend factor significantly outperforms all of the short-term reversal factor (SREV), the momentum factor (MOM), the long-term reversal factor (LREV), as well as Fama-French's market portfolio, SMB and HML factors, with the average monthly return of 1.63% and Sharpe ratio of 0.47, the trend factor also has higher alpha compared to those benchmarks and generates higher returns during the recession and financial crisis periods. In addition, HZZ claimed the trend factor is more than a combination of the SREV, MOM and LREV factors, but a unique factor that lies outside of the mean-variance frontier of those three factors.

One interesting part of HZZ's study compared to other related studies is its methodology in forming the trend factor. Firstly, HZZ utilize the historical prices rather than historical returns, which are typically used by most studies on the SREV, MOM and LREV factors, to form the portfolios. Schultz (2017) replaced HZZ's price signals with return signals and find the performance of the trend factor would become lower. Another key difference of HZZ's methodology is that, HZZ didn't include a skipping period between the portfolio formation period and the holding period, while the skipping period is widely used by most studies on the SREV, MOM and LREV factors. And the issue of the skipping period is the main topic discussed in this study.

Seemingly trivial as the few-day skipping period is, it is important for mitigating the bias from bid-ask spreads (Roll, 1984) and avoiding the opposite effects from shorter-term factors. And also, as documented by many studies, the application of a skipping period might affect the performance of the portfolio and in some cases the return of portfolio declines largely (see in section 3.3 Skipping Period). Thus it would be interesting to examine the robustness of the trend factor with the skipping period, which utilizes a more consistent method compared to other studies.

In addition, the skipping period also has strong practical senses, because of the closing price is hardly tradable. Imagine an investor whose investment strategy is based on technical analyses, for which he/she uses the closing price[1] to find the technical signals and doesn't include a skipping period (as described by HZZ), when a signal is captured, the trading rule would require the investor to buy/sell the stock with the closing price at the same time when it's observed, i.e. the moment of stock market close, in order to lock the exact price and return reflected by the theory. However, in the real world, this kind of trades are almost impossible to execute, simply because the market is closed and there is no time to take actions. Even if the investor tries his/her best to book the deals as soon as possible in the after-hour session or the next trading day, it's likely that the price has already changed, thus the return won't be exactly the same as suggested by the study.

Given the strong motivations from both academic side and practical side, it is of great interest to examine the robustness of the trend factor after applying the skipping period. Based on the evidences from previous studies, it is natural to make the hypotheses that the skipping period will reduce the return of the trend factor, and the impact is mainly through the short-term related factor.

[1] It can be the price at any moment, but this example I use closing price to follow HZZ's rule of calculating trend factor

Following the previous studies, in this study I apply two types of skipping periods: the first type (in this study called “excluding type”), which is used by many studies on short-term reversal strategies, creates the gap by excluding the most recent observations from the sample period thus the information in the gap is eliminated while no new information is taken into account; and the second type (in this study called “inserting type”), which is used by some studies on mid-term momentum and long-term reversal strategies, creates the gap by moving the whole sample period backwards, and it keeps the length of the sample period while new information is added (the ones in the beginning of changed sample period) and the information in the gap is also eliminated. And there are three lengths for each type of the skipping period tested in this study: 1-day, 5-day and 20-day length, which indicate the 1-day, 1-week and 1-month period which are widely used by many other studies as the lengths for skipping periods.

The results of this study show the return of the trend factor declines significantly after the application of skipping period. Under the first type (“excluding type”) of skipping period, the monthly return of the trend factor drops from 1.69% to 1.15% (1-day skipping period), 0.75% (5-day skipping period) and 0.68% (20-day skipping period); and under the second type (“inserting type”) of skipping period, the monthly return of the trend factor drops from 1.69% to 1.13% (1-day skipping period), 0.79% (5-day skipping period) and 0.67% (20-day skipping period). In both situations, after applying the 5- and 20-day skipping periods the return of the trend factor becomes lower than that of the SREV and MOM factor. The alpha of the trend factor also declines sharply, after applying the skipping period it becomes lower than the MOM factor’s alpha (in terms of both CAPM alpha and FF’s three factor alpha).

Following HZZ, the impacts of skipping period over the trend factor are also evaluated under the recession period and the financial crisis period, and the results indicate that the skipping period might affects the trend factor mainly through the SREV factor. Based on this, in this study the HZZ is further examined by two decomposition analysis, one is following HZZ’s Sharpe style regressions and decompose the trend factor by SREV, MOM and LREV factors, and in addition to that, the trend factor is also decomposed by 11 portfolios formed on its 11 lags.

The decomposition of trend factor suggests that out of the average 1.69% month return of trend factor, the SREV factor accounts for the largest part of 0.24% monthly return. And a closer look by decomposing the trend factor on 11 portfolios formed on its lags shows the MA5 lag accounts for 0.82% of the return. In addition, as the skipping period becomes longer, SREV's coefficient to the trend factor drops significantly. Thus as a result, it comes to the conclusion that the skipping period affects the trend factor mainly through the SREV factor.

The rest of this study is organized as follows: In the first part (Section 3. Literature Review) I provide a literature review of the past studies on the related stock return anomalies (especially the related short-term reversal effect, the momentum effect, and the long-term reversal effect), the trend factor and the skipping period; then the next part (Section 4. Hypotheses) elaborates the two hypotheses regarding the possible impacts of skipping period over the trend factor and the possible reason to be tested in the study; next section (Section 5. Data and Methodology) gives a detailed description on the data and methodology used by this study, to ensure the consistency and comparability with other studies; then (Section 6. Results and Analyses) I replicate the trend factor, and apply the skipping period of different types and lengths, I also conduct several different tests to analyze the impacts of skipping period over the trend factor; lastly (Section 7. Conclusion) is the summary of findings and conclusions.

The study examines HZZ's trend factor under the context of skipping period, the application of skipping period not only uses a more consistent method in formation the portfolios as most other studies, but also provides evidence on the trend factor's performances in a real-world setting. In addition, the analyses on decomposition of the trend factor also provides some evidence on the sources of trend factor's abnormal returns, which suggest the high returns of the trend factor is mainly from its short-term lags, which could be further explored and leveraged by future studies.

2. Literature Review

2.1. Stock Return Anomalies of Weak Form EMH

It has been well-known that the efficient market hypothesis (EMH) laid the foundation of many modern finance theories. Fama (1970) provided an early survey of market efficiency, and introduced the EMH in three forms: the weak form, the semi-strong form, and the strong form. In the weak form of EMH, the stock prices reflect all the historical trading-related information; in the semi-strong form, the stock prices not only contain the historical trading-related information, but can also efficiently adjust to all publicly available information; and in the strong form, stock prices fully reflect all available information (including also the inside information) at any time. And in a later study, Fama (1991) reviewed the recent evidences and claimed that the EMH was still largely supported by studies.

Out of the three forms of EMH, the semi-strong form of EMH is widely used as a good assumption and benchmark in a number of financial theories and studies. However, regarding the weak form of EMH, it has been discussed over the time by a large number of studies on whether it can hold true in the real-world financial market. According to Fama (1970), under the weak form of EMH, technical analysis, which utilizes the past patterns of returns to predict the future returns, should not earn abnormal returns. This is because the historical information should be included already in the stock prices, and the non-predictable stock prices should follow the pattern of random walk, where the subsequent price changes represent random departures from previous prices, with the reasoning that today’s price change only reflects today’s information and will be independent of the past information.

On the other side, however, many empirical studies reported the evidence of anomalies for the weak form of EMH, where the stock returns can somehow be predicted by the past behaviors of prices or returns. Clearly, those findings are inconsistent with Fama’s definition on the weak form of EMH, and some of the examples of the anomalies include the short-term reversal effect documented by Lehmann (1990) and Jegadeesh (1990), the momentum effect, documented by Jegadeesh and Titman (1993), the long-term reversal effect, documented by De Bondt and Thaler (1985), the seasonal and day-of-the-week effects, documented by Kerim and Ziemba (2000) and Roll (1983), and so on.

Despite that there are opponents arguing those anomalies are not really exceptions of the EMH, and the findings might lack the statistical power (Fama 1970), or the patterns didn't last long, or the findings are likely to due to data mining issues (Malkuel 2003), increasing numbers of studies are being presented to either reject the EMH or trying to explain those anomalies. An interesting piece of recent study is HZZ's trend factor, which incorporates three types of anomalies: the short-term reversal effect, momentum effect and long-term reversal effect, and according to HZZ's study, the trend factor can utilize the historical price information to generate abnormal returns, which is against the weak form of EMH. In order to better introduce the HZZ's trend factor, in the following part a review of literatures related to the short-term reversal effect, momentum and long-term reversal effect will be given first.

Firstly, the short-term reversal effect describe that over the short-term horizon, i.e. from few days up to one month, the stocks with relatively low returns in one period tend to earn higher returns in the next period. The short-term reversal effect is documented by a number of empirical studies. In an early study of Fama (1965), it was pointed out that individual stock returns have negative serial correlation; and a later study by Fama and French (1988) also documented the existence of such serial correlation; furthermore, according to the study of Lehmann (1990), portfolios formed based on previous one-week returns experienced significant return reversal in the following week, where the portfolios had positive one-week returns tend to generate -0.35% ~ -0.55% returns on average over the subsequent week, while portfolios with negative one-week returns typically to generate an average of 0.86% ~ 1.24% positive return in the next week, and such abnormal returns will persist even with adjustment for bid-ask spreads and sensible transaction costs; Jegadeesh (1990) presented the empirical results that the first-order serial correlation of monthly stock returns are highly significant, and the two extreme deciles of equally weighted portfolios based on past one-month returns led to an average monthly return of 2.49%.

Secondly, in terms of mid-term horizon, i.e. from one month up to one year, momentum effect has been reported by many studies, which describes that stocks with relatively high returns in one period tend to also have high returns in the following period. Jegadeesh and Titman (1993) documented the momentum strategy which buys stocks that performed well in the past 3- to 12- month and sell stocks with poor performance during the same period resulted in abnormal returns, with the U.S. data from 1965 to 1989, the portfolio of 6-month formation and holding period realized an annualized 12.01% return, which cannot be explained by the systematic risk.

Following the study, Jegadeesh and Titman (2001) tested the strategy with U.S. data from 1990 to 1998, and reported that momentum effect continued to exist in the 1990s. Lo and MacKinlay (1999) reported the existence of non-zero serial correlations, and many successive moves are in the same direction, which shows a pattern of momentum. In a later study by Grundy and Martin (2001), they found the momentum strategy's profitability cannot be explained by Fama-French there factor model. Outside the United States, several studies tested if momentum effect exist in other markets as well. Rouwenhorst (1998) tested the strategy with data of 12 European countries from 1985 to 1995, the results showed 1% average monthly return of the portfolio, which couldn't be explained by factors. Chui, Wei and Titman (2000) reported the momentum strategies existed in 7 Asian markets except for Japan.

Thirdly, over the long-term, i.e. more than one year, the stock returns tend to be mean reverse again. Debondt and Thaler (1985) found that strategies based on past three to five years stock returns earn around 25% returns in the following thirty six months. Fama and French (1988) documented the negative autocorrelations of stock returns more than one year in 1926-1985 sample period, finding that predictable variation accounts for about 25%-40% of the 3-5 year return variance of portfolios. Poterba and Summers (1988) also documented the negative autocorrelation of long-term stock returns by using data from the US and other 17 countries. Debondt and Thaler (1985) attributed the long-term reversal effect to investors' overreaction to market information, and the logic is that overreaction bias of investors drives the stock prices deviate from the fundamental value and then drives a mean reversion, thus a reversal strategy that buys stocks out of favor and sells stocks which returns are too high from the normal level can make profits of such behavior. While a later study by Fama and French (1996) found that the abnormal returns of the long-term reversal effect largely disappear in the three-factor model.

Overall, most of those studies related to the short-term reversal effect, the momentum and the long-term reversal effect share the following things in common: first of all, almost all of those studies are based on the historical returns; in addition, in many of the studies portfolios are formed simply by sorting the returns of the previous intervals, thus no regression techniques are required, however some studies utilize some more sophisticated and different methods, for example, the widely used Fama-French short-term factor, momentum and long-term factors (French, 2018) use double sorting methods, which also includes the market capitalization in the formation of the portfolio; furthermore, many of those studies exclude the most recent trading day(s) in the formation period, in order to mitigate the bid-ask spreads bias and to avoid the opposite effects from shorter-term factors, for example, Jagadeesh (1990) and Lehmann (1990) exclude the most recent trading day from the formation period in the short-term reversal portfolio, Jagadeesh and Titman (1993) excludes the most recent week in in the short-term reversal portfolio, French's benchmarks (available in Kenneth R. French's data library. French, 2018) also exclude a one-month period in momentum portfolio and a one-year skipping period in long-term reversal portfolio. This technique is called "the skipping period" in this study, and some more detailed descriptions will be given below in Section 2.3 "Skipping Period" .

2.2. Trend Factor

Based on the short-term reversal effect, the momentum and the long-term reversal effect, HZZ (2016) presented a trend factor which captures all the short-, mid-, and long- term price signals, in order to gain abnormal returns.

In brief, HZZ's trend factor is built in the following steps (more detailed step-by-step mathematic expressions swill be given in Section 5. "Data and Methodology"): first, the moving averages with lag lengths of 3-, 5-,10-, 20-. 50-, 100-, 200-, 400-, 600-, 800-, and 1000-day are calculated; next, those moving averages are normalized by dividing the price of the last trading day respectively; then, HZZ uses cross-section regressions to estimate the coefficients of all those normalized moving averages at month t - 1 with regard to the returns at month t ; finally, those coefficients are used to calculate the expected return of month t + 1, based on which the stocks are sorted into five portfolios, and the return difference between the highest quantile and the lowest quantile is defined as the return of the return of the trend factor.

According to the study, in the period of 1926 to 2014 the trend factor outperformed all of the short-term reversal factor, the momentum factor, the long-term reversal factor, as well as Fama-French's market portfolio, SMB and HML factors; with an average monthly return of 1.63% and Sharpe ratio of 0.47, the trend factor also generated higher returns during the recession periods and financial crisis. The study also showed the short-term information accounts for the most parts of return (around 52.2% overall, and 69.9% in recession period), and suggested the trend factor emphasizes more on the short-term information. Although the trend factor incorporates the lag lengths from 3 days to 1000 days, thus it includes all the information or price signals used by the short-term reversal, the momentum and the long-term reversal strategies, there are still two major differences.

First of all, HZZ follows the way Brock, Lakonishok and LeBaron (1992) generate price signals, so the prices of stocks are utilized to build the moving average signals and cross-section regressions are used to calculate the expected returns, while in most studies of the short-term reversal effect, momentum effect and the long-term reversal effect, returns of stocks are used to form portfolios. Secondly, HZZ utilizes normalized moving averages to generate buying or selling signals. This is quite different method compared to the related studies. Most studies on the short-term reversal effect, momentum effect and the long-term reversal effect use multiple historical returns conduct the regressions without doing any adjustment to the returns, for example, Jegadeesh (1990) constructs the model by regressing the return on month over all the prior returns. HZZ's method of using moving average, in a sense, is one way of adjusting the historical information.

HZZ argues this is because historical price has predictability over the future prices, which implies the predictability of moving averages based on price. HZZ's reasoning for this is from the empirical studies on technical analysis, for example, Treynor and Ferguson (1985), Brown and Jennings (1989) and Schwager (1989). In order to test HZZ's model from a more consistent angle, Schultz (2017) explored this issue by replacing the prices with several different form of returns (including non-normalized return, return, excess non-normalized return, excess return, non-normalized geometric mean return, geometric mean return, non-normalized geometric mean excess return and geometric mean excess return) in the formation of the trend factor, the study reports lower average monthly returns and Sharpe ratios from all the return-based trend factors compared to the price-based trend factor, while the returned-based trend factors outperform the price-based return factor during the recession periods.

Thirdly, and the most important for the topic of this study, HZZ's trend factor doesn't have the skipping period, while most of the studies related to the short-term reversal effect, the momentum and the long-term reversal effect do apply such a skipping period, with the consideration to mitigate bid-ask spread bias and avoid the opposite impacts from shorter-term factors. Much of this study will be focusing on the discussions about the skipping period, and more detailed discussions on the skipping period will be given in the following section.

2.3. Skipping Period

In this study, the skipping period is defined as one or more most recent trading day(s) which are excluded or inserted, and as a result of skipping period there will be a gap between the portfolio formation period and the portfolio holding period. Many empirical studies on the short-term reversal, momentum and long-term reversal strategies adopted a skipping period. The purpose of such a skipping period is to mitigate the potential bias from bid-ask spread, and to avoid the opposite effects from shorter-term factors.

The bid-ask spreads bias could affect the returns of strategy through observations. In the real world, stocks are traded on bid or ask prices, based on which the actual returns are calculated. However most data for empirical research only contain the midpoint of the stock price, as a result, the observed returns by studies contain measurement error to the extent of the bid-ask spreads. According to Ho and Stoll (1981), the midpoint between the market maker's bid and ask prices will deviate from the intrinsic value of the stock when the market maker is facing inventory imbalances, and it is possible that the observed return changes are price bouncing between the bid and ask prices. Roll (1984) show the bid-ask spreads will lead to the negative serial correlation of stock returns over adjacent intervals, and a skipping period between the portfolio formation period and the holding period, which excludes the last one or few trading days in the formation period, will make the return intervals not adjacent, and as a result the bias due to bid-ask spreads could be avoided.

As for the studies of mid-term momentum effect and long-term reversal effect, the skipping period can also help to avoid the opposite impacts from shorter-term factors. For example, most momentum strategies apply an one-month skipping period, in order also to avoid the short-term reversal effect within the first month after formation period. In long-term reversal strategies, the length of the skipping period applied is usually one year, in order to avoid both the short- and mid-term effects.

Based on those considerations, many studies adopt the idea of skipping period. For example, Jagadeesh (1990) build the portfolios based on several different lags of historical returns, the study also includes another group with the same sets of lags but excluding the most recent trading day, as conservative situations to compare the returns of portfolios; Lehmann (1990) build the portfolios based on previous historical return, and also build a similar set of portfolios which include a 1-day skipping period in order to mitigate the bid-ask spread bias; Jagadeesh and Titman (1993) use the strategies to select stocks based on previous 1-4 quarters returns, and use the portfolios with 1-week skipping period as a second group; in addition, the widely used momentum and long-term reversal benchmarking factors provided and updated by Kenneth R. French's data library (French, 2018) also adopt the skipping period, with the momentum factor based on the prior 2-12 month returns (i.e.1-month skipping period) to formation period, and the long-term reversal based on the prior 13-60 month returns (i.e. 1-year skipping period).

Many studies on short-term reversal use 1-day or 1-week skipping period, and usually those studies also include a set of comparison portfolios with skipping periods. One interesting finding of those studies is that returns of short-term reversal strategies decline after applying the skipping period. Jagadeesh (1990) reported after the exclusion of last trading day (1-day skipping period), the monthly returns of portfolios based on two short-term reversal related strategies (both based on historical returns) drop from 2.07% and 1.53% (without skipping period) to 1.77% and 1.08% (with 1-day skipping period) respectively; Lehmann (1990) tested the strategies based on previous 1-week, 4-week, 13-week, 26-week and 52-week returns, and there are also another sets of portfolios based on the same lag lengths excluding the most recent trading day, the results show that the application of skipping period reduces all five portfolios' return significantly, and the 1-week return based portfolio's weekly return reduces from 1.79% to 1.21%, while the 52-week return based portfolio's annual return declines from 92.89% to 62.81%. As for the reason of this trend, Jegadeesh and Titman (1995b) provided theoretical evidences that much of the short-term reversal effect could be explained by the bid-ask spreads resulting from market maker's inventory imbalances, and parts of the abnormal returns might due to the compensation for bearing inventory risks.

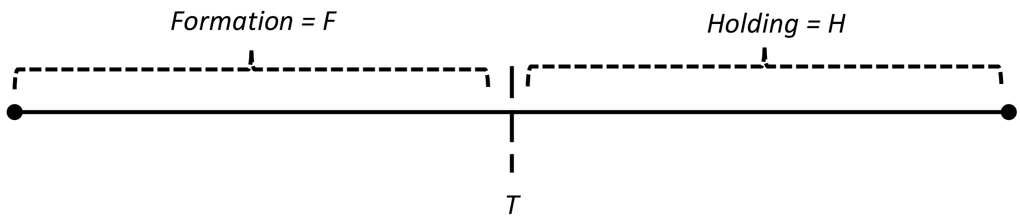
In the studies of strategies over the longer terms (momentum effect and long-term reversal effect), usually a longer length skipping period is applied. However, unlike the results from short-term reversal studies that skipping period tend to reduce the returns of portfolios, the results of those longer-term studies show less consistent impacts over the returns of portfolios. For example, in the study by Jagadeesh and Titman (1993), portfolios are formed based on quarterly historical returns, and after an one-week skipping period, all the 16 portfolios (formation period of previous 1 to 4 quarter and holding period of 1 to 4 quarter) see the returns increased slightly, while it's not clear by how much the increase is attributed to avoiding bid-ask spreads and how much is attributed to the exclusion of short-term reversal factor.

Lastly, it is important to point out that there are two types of skipping periods. The most commonly used one is the direct exclusion of the most one or several trading days from the formation period, which creates a gap between the formation and the holding period. In this study it is referred as the first type of skipping period (or the "excluding type"). Figure 1 below describes the first type of skipping period, by comparing the portfolio formation and holding period under the situations without and with a skipping period. Most studies mentioned above, such as Jegadeesh (1990) and Lehmann (1990) adopt the first type of skipping period. However, this type of skipping period clearly has a downside: when excluding the skipping period the useful information contained in that period is also excluded from the samples, thus this is used as a conservatively controlling method for potential bias (Jegadeesh, 1990).

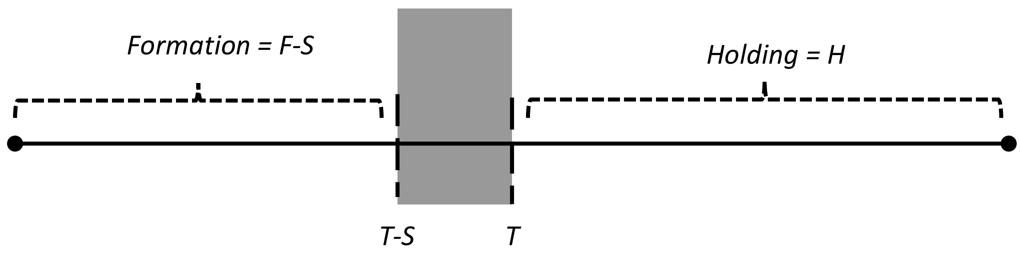
Figure 1. The first type ("excluding type") of skipping period

The figure below describes the first type ("excluding type") of skipping period, where the first sub-figure is the situation without skipping period, where the portfolio strategy includes a formation period of length F and a holding period of length H, and the second sub-figure describe the situation where the skipping period of length S is applied. Under this type of skipping period, the length of formation period becomes F-S, while the length of holding period stays as H.

Formation and holding period without skipping period:



Formation and holding period with skipping period:

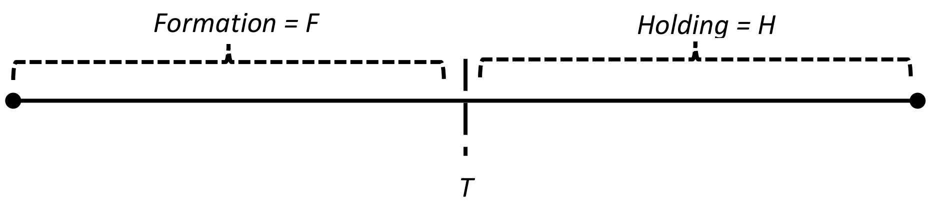


Then there is another type of skipping period, which doesn't exclude any trading days from the formation period, but creates the gap by inserting few days in between the formation period and the holding period. In this study it is referred as the second type of skipping period (or the "inserting type"). Figure 2 below describes the second type of skipping period, and it's easy to find the difference compared to the first type: the second type of skipping period doesn't exclude any useful information from the holding period, however it extends the length of the holding period. The second type of skipping period is used less frequently as the first type, but in some studies it is also used, for example, Lehmann (1990) tested the portfolio returns based on the 1-week return 2 weeks ago, 3 weeks ago, and up to 52 weeks ago, this can be considered as the second type of skipping period as it doesn't exclude any trading days from the formation period, but inserted some 2-week, 3-week and up to 52-week skipping period in between the formation and the holding period.

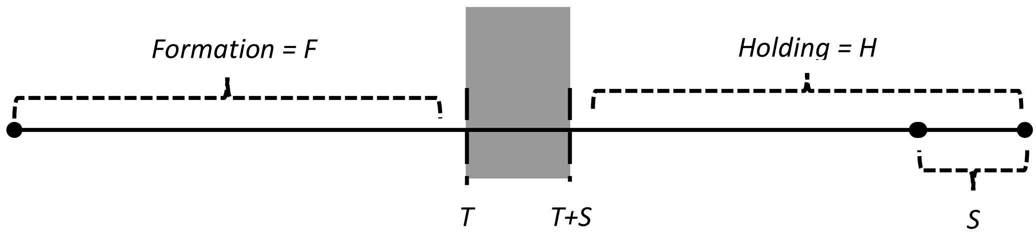
Figure 2. The second type ("inserting type") of skipping period

The figure below describes the second type ("inserting type") of skipping period, where the first sub-figure is the situation without skipping period, where the portfolio strategy includes a formation period of length F and a holding period of length H, and the second sub-figure describe the situation where the skipping period of length S is applied. Under this type of skipping period, the length of formation period stays as F, and the length of holding period stays as H, while the length of skipping period S is added on top of F+H. For example, Grundy and Martin (2001) formed the momentum strategy portfolios based on the monthly excess return over the six-month formation period from t-7 to t-2, while there is a 1-month skipping period, the length of the formation period doesn't change.

Formation and holding period without skipping period:



Formation and holding period with skipping period:



To summarize, the first type of skipping period (the "excluding type", which excludes the last observations in sample period) is more used by short-term strategies, the second type of skipping period (the "inserting type", which skips the last observations in sample period) is more often seen in mid- and long-term strategies. Thus it is important to distinguish their usages and the different rationales.

3. Hypotheses

The main purpose of this study is to provide a further examination on the performance of the trend factor with the application of skipping period. There are two major motivations behind this topic, and those two motivations are of the interests of both academical research (consistency) and practical settings (practicality).

Firstly, it would be interesting to examine the performance of trend factor without the bias of bid-ask spreads. Since the skipping period is a method widely used by other studies to mitigate the bias of bid-ask spreads and to avoid the opposite effects from shorter-term factors, thus in order to keep the results consistent and comparable, the performance of the trend factor can be and should be evaluated under the same settings, to explore whether the existence of skipping period will affect the performance of the trend factor. Secondly, in terms of practicality, real-world investors can hardly execute trades on the closing price exactly at the same time when the technical signal is observed, and most likely there is a gap between the portfolio formation period and the portfolio holding period, which suggests that a skipping period should be included.

As mentioned in the previous section “Literature Review”, many evidences from past studies suggested that the skipping period would reduce the performance of the portfolios of short-term reversal strategies, thus it is natural to make the following two hypotheses:

Hypothesis 1: The trend factor’s performance will decrease with the application of skipping period.

Hypothesis 2: The skipping period affects the performance of the trend factor mainly through the SREV factor.

In order to test the first hypothesis, in this study I first replicate HZZ’s trend factor to ensure the data and methodologies are consistent, and the results are comparable, then I apply both types of skipping periods (both the “excluding type” and the “inserting type” mention before) with different lengths to the trend factor, and compare the portfolio performance with the original trend factor from HZZ.

And in order to test the second hypothesis, it is needed to decomposition the trend factor into components with the Fama French's three factors, and to analyze the influence of each component on the overall performance of the trend factor. A more detailed description of the data and methodology will be described seen in the next section.

4. Data and Methodology

4.1. Data

In order to examine the performance of the trend factor and ensure the results are comparable with HZZ's study, in this study the same dataset is used as in HZZ's. Historical stock data from January 2, 1926 to December 31, 2014 are downloaded from the CRSP database via WRDS.

Following HZZ, two criteria are used to screen the stocks to be included in dataset: 1) the stocks must be listed on NYSE, AMEX or Nasdaq exchange (with CRSP Header Exchange Code 1, 2 or 3); 2) the stock must be ordinary common shares (with CRSP Share Code 10 or 11, this excludes closed-ends funds, RETIs, unit trusts, ADRs and foreign stocks).

Both CRSP daily stock files and monthly stock files are used in this study, where the daily data are used for calculating moving averages and price signals, and the month data are used for calculating expected returns and actual returns. Prices are adjusted for splits and dividends when necessary. At the end of every month, a price filter that excludes all the stocks with price below $5, and a size filter that excludes all the stocks in the smallest decile of NYSE breakpoints are applied to filter stocks. Overall, the CRSP data between January 2, 1926 to December 31, 2014 used by this study contains 72,222,798 observations of daily stock data, and 3,420,218 observations of monthly stock data (on the last trading day of the month).

In addition, the data of NYSE month-end breakpoints (which is used to by the size filter mentioned above), risk-free returns (Rf), and the returns of Fama-French's market portfolio (Market), size factor (SMB), value factor (HML), short-term reversal factor (SREV), momentum factor (MOM), long-term reversal factor (LREV) are from Kenneth R. French's data library (French, 2018).

For the assessment of returns during recession periods and financial crisis, this study takes the same recession periods definition as HZZ, from National Bureau of Economic Research (NBER, 2018). In the sample period from June 1930 to December 2014, there are in total 1015 months and of which 190 months are defined as recession periods and according to HZZ, December 2007 to June 2009 (a total of 19 months) is defined as the financial crisis period.

4.2. Data Processing

Since CRSP daily stock price data are not adjusted by splits and dividends while daily return data are, a price index which is calculated by compounding (1+daily return) from the first day of the stock is used in this study in replace of daily adjusted price to calculate normalized moving averages, which mathematically would lead to the same normalized moving averages mentioned by HZZ (see equation 1 below).

In the calculation of HZZ's moving average sets, it is required to have up to 1000-day continuous observations of daily data. Since missing observations widely exists in CRSP historical data, in this study NAs in daily stock returns are replaced with 0, assuming during the missing observation periods the price remained the same as the most recent valid trading day. HZZ didn't disclose their methods of handling missing data in the study.

Another issue regarding data processing is the handling of long gaps in CRSP daily stock data. Unlike the NAs mentioned before, gaps are periods of missing data not specified by CRSP (not marked with NAs). Weekends and holidays are normal gaps, while there are also longer gaps existing in the dataset due to reasons like the suspended trading of some stocks[2]. To avoid the long gaps in the calculation of moving average, I applied the rule to exclude all the price data series (for the calculation of moving average) contains at least one gap longer than 30 days. The logic is that if more than one month data is missing, the short-term reversal effect wouldn't be captured by the trend factor anymore. Again, HZZ didn't disclosed their method in the study.

According to HZZ's method, the first 1000 days (the maximum length required for moving averages) and the subsequent 12 months (the coefficients for expected return requires the averages of returns in the past 12 months) are excluded from the sample period, so there are a total of 1015 months (observations) from June 1930 to December 2014 are included in the samples. In addition, since the long-term reversal factor (LREV) is available only from January 1931, in the regressions which involves LREV, the effective sample period is from January 1931 to December 2014, a total of 1008 months (observations).

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[2] For example, after the observation of stock 10066 on the day of 2002-08-30, the next observation of the stock 10066 is on the day of 2008-01-31, during which a period of over 5 years is missing.

4.3. Results Replication with HZZ's Method

First of all, in order to check consistency of data and calculation methodology, and ensure the comparability to HZZ's results, in this study I first replicate the results of the trend factor as documented by HZZ's study.

To start with, at the end of every month t (after price and size filter are applied to exclude stocks with prices below \$5 and in the smallest decile), the normalized MAs are calculated for each stock j based on prices with lag lengths L of 3-, 5-, 10-, 20-. 50-, 100-, 200-, 400-, 600-, 800- and 1000-days. The normalized MA is defined by the following formula (equation 1 and 2 in HZZ's study):

$$\tilde{A}_{jt,L} = \frac{A_{jt,L}}{P_{j,d}^{t}} = \frac{P_{j,d-L+1}^{t} + P_{j,d-L+2}^{t} + \cdots + P_{j,d-1}^{t} + P_{j,d}^{t}}{L \times P_{j,d}^{t}} \tag{1}$$

where $\tilde{A}_{jt,L}$ is the normalized MA of stock j at end of month t with lag length L, $A_{jt,L}$ is the moving average of stock j at the end of month t with lag length L, and $P_{j,d}^{t}$ is the closing price for stock j at the last trading day d of month t.

Secondly, at the end of each month t, a cross-section regression is used to calculate the coefficients of each lag-specific normalized MA with regard to the monthly return (equation 3 in HZZ's study), where $r_{j,t}$ is the return of stock j in month t, $\beta_{i,t}$ is the coefficient of the normalized MA with leg L_i in month t, and $\beta_{0,t}$ is the intercept in month t.

$$r_{j,t} = \beta_{0,t} + \sum_{i} \beta_{i,t}\, \tilde{A}_{jt-1,L_i} + \varepsilon_{j,t}\,, \qquad j = 1, \dots, n \tag{2}$$

Thirdly, at the end of each month t, the coefficients of month t and prior 11 months are used to estimate the coefficients of month t+1 (equation 5 in HZZ's study):

$$E_t[\beta_{i,t+1}] = \frac{1}{12} \sum_{m=1}^{12} \beta_{i,t+1-m} \tag{3}$$

Fourthly, those expected coefficients and the normalized MAs at the end of month t are used to calculate the expected returns of month t+1 (equation 4 in HZZ's study):

$$E_t[r_{j,t+1}] = \sum_i E_t[\beta_{i,t+1}]\ \tilde{A}_{jt,L_i} \tag{4}$$

Lastly, the monthly-rebalanced portfolio strategy is defined as follows: at the end of each month, all the stocks are sorted by their expected returns into five equal-weighted portfolios, and stocks in the highest quantile are defined as "winners" and lowest quantiles as "losers". The difference between the returns of "winners" and the "losers" are defined as the returns of trend factor.

4.4. Applying Skipping Period into Trend Factor

After the replication of HZZ's trend factor, the next step is to apply the two types of skipping periods into the trend factor, in order to test the Hypothesis 1.

As for the lengths of the skipping periods, I include 1-, 5- and 20-day periods, which indicate 1-day, 1-week, and 1-month lengths. Those lengths are commonly used by previous studies: for example, Lehmann (1990) and Jegadeesh (1990) applied 1-day skipping period (of the first type) in short-term reversal strategies, Jegadeesh and Titman (1993) applied 1-week period (of the first type) in momentum strategies, Grundy and Martin (2001) applied 1-month period (of the second type) in momentum strategies.

By applying the first type "excluding type") skipping period with length S1, the most recent S1 trading days are excluded from the observations in formation period and the moving averages are shortened accordingly. For example, when S1= 1, the moving averages would become 2-, 4-, 9-, 19-, 49-, 99-, 199-, 399-, 599-, 799-, and 999- days.

And when the second type ("inserting type") skipping period with length S2 is applied, the moving averages stay the same length while the starting day and ending day move backwards accordingly. For example, the 3-day moving average captures the price at month end (day d), one day before (day d-1) and two days before (day d-2), and when S2=1 is applied, the 3-day moving average will capture the price of day d-1, d-2 and d-3 instead.

Then I conduct several similar tests as HZZ to examine the performance of the trend factor with those skipping periods. Firstly the summary statistics (where mean, standard deviation, Sharpe ratio, skewness and excess kurtosis are calculated), as well as the performance during the recession and financial crisis periods are presented as a general overview of trend factor with skipping period; next, the alpha of the trend factors with skipping periods is calculated, to presented a detailed view on the changes of its abnormal return.

4.5. Determining the Key Driver for Performance Change

If the Hypothesis 1 is supported (i.e. the performance of trend factor declines with the application of trend factor) by the analyses mentioned above, then the next step would be to test the Hypothesis 2 and further analyze which of the short-end, mid-end and long-end components drive such decline.

HZZ conducted the Sharpe style regressions in order to determine the contribution of the short-term reversal factor, the momentum factor and the long-term reversal factor to the return of the trend factor. The Sharpe Style regression (Sharpe, 1980) is used to determine the contribution of various sub-portfolios to the overall fund performance, and it puts constraints over the coefficients where all the coefficients cannot be negative and the sum must equal to 1. Following HZZ, I conduct the following regression to identify the contribution of the SREV, MOM and LREV factors to the trend factor:

$$r_{Trend,t} = \alpha + \beta_1 r_{SREV,t} + \beta_2 r_{MOM,t} + \beta_3 r_{LREV,t} + \epsilon_t \tag{5}$$

where

$$\beta_1 \geq 0, \qquad \beta_2 \geq 0, \qquad \beta_3 \geq 0, \qquad and\ \beta_1 + \beta_2 + \beta_3 = 1$$

According to the results of the Sharpe style regression, the coefficients $\beta_1, \beta_2, \beta_3$ determine the sensitivity of the trend factor’s return to the returns of SREV, MOM and LREV factors, i.e. how much movement of the trend factor’ return is due to the movement of the SREV, MOM and LREV factors’ returns. By applying skipping periods of different types and lengths into the trend factor, it is possible to find how those coefficient changes with the skipping period. In addition, in order to get a full picture of each factor’s impact over the trend factor, it’s also important to take the returns of the factors $r_{SREV,t}$, $r_{MOM,t}$,$r_{LREV,t}$ into the consideration.

Given the values of $r_{SREV,t}$, $r_{MOM,t}$, $r_{LREV,t}$ are known, it's easy to multiply each factor's return with its coefficient, and get a "starting point" for analysis (in this study its' called the contributed return). The analysis for contributed return will be helpful to identify the key driver, by excluding the factors with high coefficient but low contributed returns.

The Sharpe style regressions mentioned above could help to identify which of the SREV, MOM and LREV factor is the key driver for the trend factor's performance when skipping periods are applied. In addition to that, a more detailed analysis would be to regress the trend factor's return over the 11 portfolios formed based on individual lags: First is to construct the 11 single lag portfolios in similar way as the trend factor is constructed, and get the basic summary statistics of those portfolios; then similar to the previous analyses on the SREV, MOM and LREV factors, the coefficients of the 11 portfolios and the contributed returns of those 11 portfolios are calculated, in order to identify the key driver.

Those methods mentioned above will help to identify the key driver behind the trend factor's performance change when the skipping periods by firstly analyzing under the scope of the SREV, MOM and LREV factors, and then further narrowing down into each of the trend factor's 11 lags. Finally, the results will help to answer whether the Hypothesis 2 is true.

5. Results and Analyses

5.1. Replicated Results

Based on the data and methods described in section 5, the trend factor is replicated in the beginning to ensure the comparability and consistency between this study and HZZ's original study. Table 1 shows the summary statistics of HZZ's original trend factor, the replicated trend factor, as well as the calculated benchmarking factors included in HZZ's study using same data sources as mentioned above. The original summary statistics from HZZ's study can be found in Appendix 1.

Overall, the characteristics of the replicated trend factor is very close to HZZ's original results, though slight discrepancies exist: the replicated trend factor has the monthly average return of 1.69% (compared to 1.63% in HZZ's results), the standard deviation of 4.02% (compared to 3.45% in HZZ's results), and the Sharpe ratio, skewness and excess kurtosis are 0.42 (compared to 0.47 in HZZ's results), 1.50 (compared to 1.47 in HZZ's results), and 19.77 (compared to 11.3 in HZZ's results) respectively.

Despite the slight discrepancies between two studies, the replicated results support HZZ's findings that, in terms of monthly average return and the Sharpe ratio, the trend factor significantly outperforms the benchmarking factors, i.e. the short-term reversal factor (SREV), momentum factor (MOM), long-term reversal factor (LREV), and the Fama-French market portfolio (Market), size factor (SMB), and value factor (HML). The replicated results also shows very similar skewness to HZZ's original results and a large excess kurtosis, indicating the distribution of monthly returns has a fat right tail - the same shape as HZZ's results. In addition to that, the results of benchmarking factors calculated in this study are almost the same as HZZ's results, except for few very tiny discrepancies which can be neglected, this also indicate the consistency with HZZ's study.

Those discrepancies can possibly be explained by two issues. Firstly, HZZ might pre-filter the historical data in a slight different way especially when dealing with the NA and long gaps of CRSP data (as mentioned in section 5.2 Data Processing, HZZ didn't explain their methodology to handle those two issues); Secondly, HZZ's calculations were based on the CRSP data downloaded in 2015, and the replicated results are based on the CRSP data downloaded in 2018, the changes in data source (as CRSP frequently update database[3]) might lead to some discrepancies between the results, and a good proof for this is there are few neglectable discrepancies on some summary statistics of LREV, Market, SMB, and HML.

Based on the observations above, it is sufficient to conclude that the data and methodology used in this study are largely consistent with HZZ's original study, and the replicated results are comparable to HZZ's original results, though we should notice the existence of tiny discrepancies. Thus, the replicated results will be used as a "starting point" in this study, and the following analyses of this study will be built on the comparisons between the trend factor with skipping periods and the replicated trend factor.

Table 1. The original and replicated trend factor: summary statistics.

The table reports the summary statistics, including mean, standard deviation, Sharpe ratio, skewness and excess kurtosis, of HZZ's results (Trend-HZZ), the replicated results by this study in 2018 (Trend-R), and other benchmark factors used by HZZ, using the latest data from Ken French's data library (French, 2018), including the short-term reversal factor (SREV), the momentum factor (MOM), the long-term reversal factor (LREV), and Fama-French's market portfolio (Market), SMB and HML factors. A total of 1015 months (observations) are included in the sample period, from June 1930 to December 2014. The t-statistics are in parentheses and significance at 1% level is given by ***.

Factor	Mean (%)	Std. dev (%)	Sharpe ratio	Skewness	Excess kurtosis
Trend-HZZ	1.63*** (15.0)	3.45	0.47	1.47	11.3
Trend-R	1.69*** (13.41)	4.02	0.42	1.50	19.77

[3] HZZ's paper was firstly received by the Journal of Financial Economics on 14 January 2015, and a revision was made on 28 September 2015. As mentioned in Kenneth R. French's website (French, 2018), several CRSP updates have resulted in historical return changes especially in the early years.

SREV	0.79*** (7.21)	3.49	0.23	0.99	8.18
MOM	0.79*** (3.26)	7.69	0.10	-4.41	40.42
LREV	0.34*** (3.09)	3.50	0.10	2.93	24.76
Market	0.62*** (3.69)	5.40	0.12	0.27	7.98
SMB	0.26*** (2.57)	3.24	0.08	2.02	19.82
HML	0.41*** (3.70)	3.56	0.12	2.19	19.10

5.2. Summary Statistics

Table 2 summarizes the results after applying both types of skipping periods of 1-, 5- and 20-day lengths, and the comparison with benchmarking factors SREV, MOM, LREV, Market, SMB and HML. The results well support the Hypothesis 1, as the return of the trend factor largely declines when longer skipping periods applied, and eventually after applying the 5- and 20-day skipping periods the return of the trend factor becomes lower than that of the SREV and MOM factor.

The results suggest two types of skipping periods show similar impacts on the trend factor. With the 1-day skipping periods, the average monthly average returns significantly decline from 1.69% to 1.16% and 1.13% (with respect to two types of skipping period). And after applying the 5-day skipping periods, the returns further reduce to 0.75% and 0.79%, which are below the returns of SREV (0.79%) and MOM (0.79%). While after applying the 20-day skipping periods, the monthly average returns are only 0.68% and 0.67%, indicating that without the short-term reversal effect, the trend factor's return would have been lower than that of the SREV (0.79%), MOM (0.79%) and Market (0.62%) factors.

However, the results also show that despite the return of the trend factor declines with the application of skipping periods, the standard deviation remains stable between 3.99% (the lowest, from the trend factor with 20-day “inserting type” skipping period, i.e. S2=20) and 4.31% (the highest, from the trend factor with 5-day “excluding type” skipping period, i.e. S1=5). As a result of that, even though the Sharpe ratio of the trend factor also declines with the application of skipping periods, it doesn’t go down as much significant as monthly average return. Even with the 20-day skipping periods, the Sharpe ratios are 0.16 and 0.17, which are lower than the Sharpe ratio of SREV (0.23), but still higher than that of the MOM (0.10), LREV (0.10), Market (0.12), SMB (0.08) and HML (0.12).

In addition, the skewness of the trend factor become lower with skipping periods, though it didn’t show a clear pattern with the length of skipping period, still it suggests the skipping periods would distort the distribution of the monthly returns and reduce its fat right tail, which indicates lower probability of high returns. While in terms of the excess kurtosis, except for the trend factors with the 1-day “excluding type” skipping period (S1=1, 19.99) and the 5-day “inserting type” skipping period (S2=5, 25.61), all the excess kurtosis values become lower, which also suggest the skipping periods reduce the fat right tail of return distribution.

To summary, the performance of the trend factor, in terms of all the major indicators including mean, Sharpe ratio, skewness and excess kurtosis declines with the applications of the skipping period. The return of the trend factor significantly declines as the longer lag length of the skipping periods applied, and eventually the superiority over benchmarking factors disappears when skipping periods of 5- and 10- day lengths applied.

Table 2. The trend factor with skipping period: summary statistics

This table provides the same summary statistics as Table 1, including mean, standard deviation, Sharpe ratio, skewness and excess kurtosis, of the trend factor, as well as the trend factors with both types of 1-, 5- and 20-day skipping periods (where S1 is the length of first type, i.e. “excluding type” skipping period which excludes the most recent observations of the formation period, and S2 is the length of second type, i.e. “inserting type” skipping period which doesn’t exclude the most recent observations but insert a gap after formation period), in comparison to the short-term reversal factor (SREV), the momentum factor (MOM), the long-term reversal factor (LREV), and Fama-French's market portfolio (Market), SMB and HML factors. The t-statistics are in parentheses and significance at 1% level is given by ***.

Factor	Mean (%)	Std. dev (%)	Sharpe ratio	Skewness	Excess kurtosis
Trend	1.69*** (13.41)	4.02	0.42	1.50	19.77
Trend (S1=1)	1.16*** (8.80)	4.18	0.28	1.01	19.99
Trend (S1=5)	0.75*** (5.83)	4.12	0.18	0.79	18.35
Trend (S1=20)	0.68*** (5.05)	4.31	0.16	0.66	15.42
Trend (S2=1)	1.13*** (8.65)	4.16	0.27	0.55	18.71
Trend (S2=5)	0.79*** (6.02)	4.18	0.19	1.44	25.61
Trend (S2=20)	0.67*** (5.34)	3.99	0.17	0.86	15.66
SREV	0.79*** (7.21)	3.49	0.23	0.99	8.18
MOM	0.79*** (3.26)	7.69	0.10	-4.41	40.42
LREV	0.34*** (3.09)	3.50	0.10	2.93	24.76
Market	0.62*** (3.69)	5.40	0.12	0.27	7.98
SMB	0.26*** (2.57)	3.24	0.08	2.02	19.82
HML	0.41*** (3.70)	3.56	0.12	2.19	19.10

5.3. Performance in Recession and Financial Crisis Periods

HZZ also evaluated the performance of the trend factor during the recession periods and the financial crisis periods, thus the same tests are included in this study to explore the effects of skipping period over portfolio returns during such periods. In this study the same definition of recession and the financial crisis periods is used, which is the same as HZZ and from the National Bureau of Economic Research (NBER, 2018) , and the results are reported in Table 3.

Panel A of Table 3 reports the summary statistics of all the factors' performance during all recession periods. First of all, consistent with HZZ's study, both the return and volatility of the trend factor become higher during recession periods compared to the normal period, the average monthly return increases from 1.69% to 2.28%, and standard deviation increases from 4.02 to 5.48, resulting the same Sharpe ratio of 0.42. The results show that the trend factor outperforms the SREV, MOM, LREV, Market, SMB and HML factors, with the highest average monthly return (2.28%) and Sharpe ratio (0.42). However, after applying two types of skipping periods, the trend factor sees significant declines in its performance. With the 1-day skipping periods applied, the average monthly return jumps from 2.28% to 1.51% and 1.43% (with respect to two types of skipping periods), and the Sharpe ratio declines from 0.42 to 0.28 and 0.26 (with respect to two types of skipping periods); with the 5-day and 20-day skipping periods applied, the performance of the trend factor further declines, and underperforms the SREV factor in both monthly average return and Sharpe ratio. The results are clear evidences to support the Hypothesis 1.

Panel B of Table 3 reports the summary statistics of all the factors' performance during the most recent financial crisis periods. The results show that the trend factor outperforms other factors with a 0.92% monthly average return and 0.14 Sharpe ratio, which is consistent with HZZ's findings that the trend factor during financial crisis periods has weaker performance than it's in recession period, but still outperforms the benchmarking factors except that SMB factor has a higher Sharpe ratio, though it should be also noted that the results are not statistically significantly (which is also consistent with HZZ's original study). Interestingly, as opposite in recession periods, both types of the skipping periods seems to improve the performance of trend factor. With the 1-day skipping periods, the average monthly return increases from 0.92% to 1.34% and 1.23% (with respect to two types of skipping periods), and the Sharpe ratio rises from 0.14 to 0.18 and 0.18 (with respect to two types of skipping periods); And the 20-day “excluding type” skipping period (i.e. S1=20) leads to the best performance of the trend factor, with average monthly return of 1.53% and Sharpe ratio of 0.21. However, again those results are not statistically significant, thus it’s not sufficient to conclude whether the results in financial crisis periods supports the Hypothesis 1.

Apart from those, when compare the results of the performances in recessions periods and the financial crisis periods, there seems to be a pattern. The results in Table 3 - Panel A also show the SREV factor outperforms the MOM and LREV factors with its 1.20% average monthly return and 0.22 Sharpe ratio. It implies that the performance of the trend factor might largely due to the SREV factor, as the results show that with the increasing length of skipping periods, the performance of the trend factor declines more. This relationship is also suggested by the results of financial crisis periods, as shown in Table 3 - Panel B, the SREV has a -0.79% average monthly return, which is lower than that of the trend factor, which is 0.92%, this indicates that SREV factor actually drags the performance of trend factor, and when the skipping periods remove parts of SREV’s effects, the trend factor shows a stronger performance. However, despite the statistical insignificance during the financial crisis periods, it’s also hard to distinguish if such effects are from SREV, MOM or LREV during the financial crisis periods, and more evidences are needed to test the Hypothesis 2.

Table 3. The trend factor with skipping period: recession periods

This table provides the same summary statistics as Table 1, including mean, standard deviation, Sharpe ratio, skewness and excess kurtosis, of the trend factor, as well as the trend factors with both types of 1-, 5- and 20-day skipping periods (where S1 is the length of first type, i.e. “excluding type” skipping period which excludes the most recent observations of the formation period, and S2 is the length of second type, i.e. “inserting type” skipping period which doesn’t exclude the most recent observations but insert a gap after formation period), in comparison to the short-term reversal factor (SREV), the momentum factor (MOM), the long-term reversal factor (LREV), and Fama-French’s market portfolio (Market), SMB and HML factors, under the recession periods and financial crisis periods. The t-statistics are in parentheses and significance at 1% level is given by ***.

Factor	Mean (%)	Std. dev (%)	Sharpe ratio	Skewness	Excess kurtosis
	Panel A: Recession periods				
Trend	2.28*** (5.73)	5.48	0.42	0.52	4.87
Trend (S1=1)	1.51*** (3.83)	5.43	0.28	-0.23	5.07
Trend (S1=5)	0.76 (1.98)	5.27	0.14	-1.05	8.99
Trend (S1=20)	0.71 (1.82)	5.42	0.13	-0.71	8.35
Trend (S2=1)	1.43*** (3.54)	5.55	0.26	-0.81	6.45
Trend (S2=5)	0.99* (2.62)	5.21	0.19	-1.00	9.65
Trend (S2=20)	0.67* (1.85)	4.98	0.13	-1.23	5.61
SREV	1.20*** (3.07)	5.39	0.22	0.84	3.24
MOM	0.20 (0.24)	11.46	0.02	-3.17	17.11
LREV	0.48 (1.58)	4.15	0.12	1.23	6.02
Market	-0.67 (-1.13)	8.24	-0.08	0.50	3.77
SMB	0.01 (0.06)	3.30	0.00	0.55	2.00
HML	0.17 (0.45)	5.17	0.03	2.85	18.38

Factor	Mean (%)	Std. dev (%)	Sharpe ratio	Skewness	Excess kurtosis
	Panel B: Financial crisis periods				
Trend	0.92 (0.61)	6.57	0.14	0.31	-0.54
Trend (S1=1)	1.34 (0.81)	7.25	0.18	0.78	0.35
Trend (S1=5)	0.98 (0.73)	5.85	0.17	0.13	-0.62
Trend (S1=20)	1.53 (0.91)	7.37	0.21	1.09	1.61
Trend (S2=1)	1.23 (0.79)	6.80	0.18	0.76	0.41
Trend (S2=5)	1.05 (0.86)	5.3	0.20	-0.08	-0.77
Trend (S2=20)	0.71 (0.61)	5.04	0.14	-0.07	-0.88
SREV	-0.79 (-0.61)	5.65	-0.14	-0.10	-1.14
MOM	-3.89 (-1.27)	13.39	-0.29	-1.29	0.98
LREV	0.01 (0.02)	3.72	0.00	0.12	-0.45
Market	-2.02 (-1.25)	7.07	-0.29	-0.19	-0.48
SMB	0.59 (1.14)	2.27	0.26	0.23	-1.00
HML	-0.54 (-0.53)	4.49	-0.12	-0.45	0.04

5.4. Alpha

Following HZZ, in this study I also analyze the alphas and risk loadings, with regard to the CAPM and the Fama French three-factor model, of the trend factor as well as the trend factor with skipping periods, the momentum factor is also included in the analysis following HZZ. The results are reported in Table 4 and clearly support the Hypothesis 1.

Table 4 shows that with longer skipping periods, both CAPM alpha and Fama-French three-factor alpha of the trend factor decrease significantly. After the 1-day skipping period, the trend factor's CAPM alpha becomes 0.81% and 0.79% (with regard to the two types of skipping periods) from 1.33%, and the Fama-French three-factor alpha becomes 0.78% and 0.77% (with regard to the two types of skipping periods) from 1.32%, both are lower than those of the momentum factor (MOM), of which the CAPM alpha is 1.07% and the Fama-French three-factor alpha is 1.36%. And alphas of the trend factor further decline when the length of skipping periods becomes longer.

In addition to that, when looking at the Fama-French three-factor risk loadings, the market factor plays less role in the return of the trend factor while the size factor (SMB) contributes to increasingly larger parts of the return with the longer skipping periods applied.

Table 4. CAPM and Fama-French alphas.

The table reports the Jensen's alpha and risk loadings with respect to the CAPM and the Fama-French three-factor model. The trend factor, as well as the trend factors with 1-, 5- and 20-day skipping periods (where S1 is the length of first type, i.e. "excluding type" skipping period which excludes the most recent observations of the formation period, and S2 is the length of second type, i.e. "inserting type" skipping period which doesn't exclude the most recent observations but insert a gap after formation period) are included, in comparison to the momentum factor (MOM). The t-statistics are in parentheses and significance at 1% level is given by ***, 5% level by **, and 10% level by *.

	Panel A: CAPM		Panel B: Fama-French			
	α (%)	β_{mkt}	α (%)	β_{mkt}	β_{smb}	β_{hml}
Trend	1.33***	0.13***	1.32***	0.10***	0.19***	-0.06
	(10.59)	(5.57)	(10.58)	(4.01)	(4.77)	(-1.55)
Trend (S1=1)	0.81***	0.10***	0.78***	0.05*	0.31***	-0.06
	(6.16)	(4.19)	(6.11)	(1.94)	(7.49)	(-1.63)
Trend (S1=5)	0.42***	0.08***	0.37***	0.01	0.38***	-0.01
	(3.23)	(3.51)	(2.96)	(0.43)	(9.28)	(-0.30)
Trend (S1=20)	0.35**	0.08***	0.30**	0.01	0.37***	-0.02
	(2.56)	(3.31)	(2.30)	(0.46)	(8.67)	(-0.45)
Trend (S2=1)	0.79***	0.10***	0.77***	0.05*	0.31***	-0.09**
	(6.02)	(3.97)	(6.03)	(1.87)	(7.59)	(-2.32)
Trend (S2=5)	0.45***	0.10***	0.41**	0.03	0.33***	-0.02
	(3.40)	(3.95)	(3.19)	(1.36)	(7.81)	(-0.66)
Trend (S2=20)	0.33*	7.24**	0.30**	0.02	0.28***	-0.00
	(2.70)	(3.11)	(2.44)	(0.65)	(7.12)	(-0.07)
MOM	1.07***	-0.45***	1.36***	-0.23***	-0.47***	-0.76***
	(4.62)	(-10.61)	(6.47)	(-5.35)	(-6.91)	(-12.55)

5.5. Sharpe Style Regressions

The previous analyses and results in Section 6.2-6.4 provide evidences to support the Hypothesis 1, and in addition to that, the Section 6.3 also implies the change of trend factor's performance might be somewhat connected to the SREV factor, though the results are not statistically significant and the evidences are not sufficient.

In order to provided further results to test the Hypothesis 2, in this subsection, I followed HZZ and utilized the Sharpe Style regressions to explore how the performance of the trend factor is related to SREV, MOM and LREV factors, and also examine the changes of such relationship across different lengths of skipping periods. The Sharpe Style regression (Sharpe, 1980) is used to determine the contribution of various sub-portfolios to the overall fund performance, and it puts constraints over the coefficients where all the coefficients cannot be negative and the sum must equal to 1. Following HZZ, I conducted the following regression to identify the contribution of the SREV, MOM and LREV factors to the trend factor:

$$r_{Trend,t} = \alpha + \beta_1 r_{SREV,t} + \beta_2 r_{MOM,t} + \beta_3 r_{LREV,t} + \epsilon_t \tag{5}$$

where

$$\beta_1 \geq 0, \qquad \beta_2 \geq 0, \qquad \beta_3 \geq 0, \qquad and\ \ \beta_1 + \beta_2 + \beta_3 = 1$$

in which the monthly returns of the trend factor are regressed over the monthly returns of the short-term reversal factor (SREV), the momentum factor (MOM) and the long-term reversal factor (LREV), and t is the month. Mathematically, the coefficient β_i measures the contribution of the movement on each factor's monthly return over the movement of the trend factor's monthly return.

In addition to the regression over the trend factor, similar regressions are also conducted on the trend factor with both types of skipping periods of 1-, 5- and 20-day lengths. And in addition to those, the regressions are further split by whole sample period, recession periods and expansion periods, to evaluate the contributions of the SREV, MOM and LREV factors in different economic conditions. It should be pointed out that HZZ included the period from June 1930 to December 2014 in the regression analysis, while in this study the sample period is from January 1931 to December 2014. This is because the LREV factor data is only available from January 1931 in Kenneth R. French's data library (French, 2018), while HZZ didn't explain the data source for LREV factor before January 1931.

The results of the Sharpe style regressions are reported in Table 5. The results show that, without any skipping period, the LREV factor accounts for the most movement of the trend factor's return in the whole sample period (48.46%) and in expansion period (61.30%), while in recession period it is mainly contributed by the SREV factor (56.49%). This is somewhat different from the original HZZ's results, where the SREV accounts for the most in all three periods. This difference might be from the different length of the sample period, as mentioned before, however this difference in the starting point is not crucial for our analysis on the impacts of skipping period on the trend factor, as the focus of this study is on the changes of three factor's contributors when applying different skipping periods, which will be described in details in the following part.

The results in Table 5 shows that, with the application of both types of skipping periods, the SREV factor's contribution in the whole sample period decreases from 30.22% (no skipping period) to 20.37% and 21.03% (with respect to 1 day skipping period of both types), 9.60% and 8.14% (with respect to 5 day skipping period of both types), 1.20% and 10.07% (with respect to 20 day skipping period of both types). The same declining pattern is also clearly observed in recession and expansion periods. Where in the recession periods, the SREV factor's contribution decreases from 56.49% (no skipping period) to 45.72% and 44.85% (with respect to 1 day skipping period of both types), 25.52% and 30.92% (with respect to 5 day skipping period of both types), 14.34% and 27.23% (with respect to 20 day skipping period of both types); and in the expansion periods, the SREV factor's contribution in the decreases from 13.19% (no skipping period) to 4.12% and 5.85% (with respect to 1 day skipping period of both types), -0.41% and -0.06% (with respect to 5 day skipping period of both types), -7.22% and -0.01% (with respect to 20 day skipping period of both types). The results also show that the contribution of LREV factor increases as the decline of the contribution of the SREV factor, while MOM's contribution almost stay unchanged.

Despite the difference with HZZ's study in the starting point, the comparison of results with different skipping periods support the Hypothesis 2 that skipping periods affect the performance of the trend factor mainly through SREV factor.

Table 5. Sharpe Style regressions

The table reports the results of Sharpe Style regressions, regressing the returns of the trend factors and the trend factors with 1-, 5- and 20-day skipping periods (where S1 is the length of first type, i.e. "excluding type" skipping period which excludes the most recent observations of the formation period, and S2 is the length of second type, i.e. "inserting type" skipping period which doesn't exclude the most recent observations but insert a gap after formation period), on the returns of the short-term reversal factor (SREV), the momentum factor (MOM), and the long-term reversal factor (LREV). The coefficients are constrained to be positive and their sum is equal to 1. The sample period is from January 1931 to December 2014, including 1008 months (observations). The t-statistics are in parentheses and significance at 1% level is given by ***, 5% level by **, and 10% level by *.

Trend	Whole sample period	Recession period	Expansion period
SREV	30.22***	56.49***	13.19***
	(11.41)	(3.78)	(4.56)
MOM	21.32***	15.38***	25.51***
	(15.55)	(5.13)	(16.97)
LREV	48.46***	28.13***	61.30***
	(18.98)	(5.00)	(22.05)

Trend (S1=1)	Whole sample period	Recession period	Expansion period
SREV	20.37***	45.72***	4.12***
	(7.49)	(7.63)	(1.38)
MOM	24.19***	20.94***	26.57***
	(17.16)	(6.78)	(17.05)
LREV	55.44***	33.34***	69.31***
	(21.14)	(5.76)	(24.07)

Trend (S1=5)	Whole sample period	Recession period	Expansion period
SREV	9.60***	25.52***	-0.41
	(3.52)	(4.11)	(-0.14)
MOM	25.06***	24.57***	25.48***
	(17.75)	(7.67)	(16.18)
LREV	65.34***	49.91***	74.93***
	(24.85)	(8.30)	(25.74)

Trend (S1=20)	Whole sample period	Recession period	Expansion period
SREV	1.20*	14.34**	-7.22**
	(0.43)	(2.19)	(-2.32)
MOM	28.00***	24.65***	30.27***
	(19.21)	(7.30)	(18.72)
LREV	70.81***	61.01***	76.96***
	(26.10)	(9.64)	(25.74)

Trend (S2=1)	Whole sample period	Recession period	Expansion period
SREV	21.03***	44.85***	5.85**
	(7.89)	(7.64)	(1.99)
MOM	25.15***	23.52***	26.45***
	(18.23)	(7.77)	(17.30)
LREV	53.82***	31.62***	67.70***
	(20.96)	(5.57)	(23.95)

Trend (S2=5)	Whole sample period	Recession period	Expansion period
SREV	8.14**	30.92***	-0.06**
	(2.43)	(4.82)	(-2.08)
MOM	24.62***	23.16***	25.78***
	(17.00)	(7.00)	(16.31)
LREV	67.24***	45.92***	80.55***
	(24.94)	(7.41)	(27.57)

Trend (S2=20)	Whole sample period	Recession period	Expansion period
SREV	10.07	27.23***	-0.01*
	(1.49)	(4.83)	(1.66)
MOM	26.49***	25.88***	27.00***
	(19.48)	(8.90)	(17.32)
LREV	63.44***	46.89***	73.73***
	(25.05)	(8.60)	(25.59)

5.6. Contributed Returns of the SREV, MOM and LREV Factors

The previous Sharpe style regressions primarily focus on the coefficient of the SREV, MOM and LREV factors' return over the trend factor's return, which finds that with the skipping periods would reduce SREV's contribution to the movement of the trend factor's return. Another interesting perspective is to look at the three factor's contribution to the trend factor's overall return (rather than the movement of the return), by multiplying the monthly return of each factor with its coefficient.

Table 6 reports the results of the coefficients, the average monthly returns, and the contributed returns which are multiplied by the former two values, of the SREV, MOM and LREV factors. It should be pointed out that here the regression is slightly different from the previous Sharpe style regressions, because the constraints in equation (5) are removed in order to reflect the actual correlations, while the Sharpe style regression's constraints are more suitable for analyses of contributions, however mathematically both ways should lead to similar results. The regression show that the coefficients of the SREV, MOM and LREV factors with regard to the trend factor are 0.16, 0.15 and 0.33, similar to the contribution ratios from Sharpe style regressions 30.22%, 21,32% and 48.46.

The results of contributed returns show that out of the 1.69% total average monthly return of the trend factor, 0.24% are contributed by the SREV factor, 0.17% are due to MOM factor and

0.16% are from the LREV factor. Thus, during the whole sample period, SREV factor accounts for the largest part of the trend factor's return compared to MOM and LREV factors .

Combining the results of Sharpe style regressions (Table 5) and the contributed returns (Table 6) gives us a more comprehensive view on how the skipping periods affect the trend factor through the SREV factor: Firstly, without any skipping periods, the SREV factor accounts for the largest part of the trend factor's contributed return, as the commonly used skipping periods are within one-month length, it affects mostly the short-term information contained in the trend factor. For example, when the 20-day skipping period of the first type ("excluding type") is applied, it will eliminate the whole SREV factor, and 0.24% out of the 1.69% return would be removed. Secondly, as shown by the Sharpe style regressions, the SREV factor's coefficient to the trend factor will drop significantly as the length of skipping period increases, this will further reduce the trend factor's return on top of the first reason. These findings not only well support the Hypothesis 2 that the skipping periods affects the trend factor mainly though the SREV factor, but also provide detailed explanations on how this happens.

Table 6. Contributed returns of the SREV, MOM and LREV factors

The table reports the contributed monthly average return of the short-term reversal factor (SREV), the momentum factor (MOM), and the long-term reversal factor (LREV), to the overall monthly average return (1.69%) of the trend factor. The contributed return of each factor is multiplied by its coefficient (whole sample) from the Sharpe Style regressions, and the average monthly return of the factor. The t-statistics of the coefficient and the average monthly return are in parentheses and significance at 1% level is given by ***, 5% level by **, and 10% level by *.

Factor	Coefficient (whole sample)	Average monthly return (%)	Contributed return (%)
SREV	0.16***	0.79***	0.24
	(4.57)	(7.21)	
MOM	0.15***	0.79***	0.17
	(9.34)	(3.26)	

LREV	0.33***	0.34***	0.16
	(9.20)	(3.09)	

5.7. Decomposition of the Trend Factor by Single Lag Portfolios

The results of Sharpe style regressions in Section 6.5 provide evidences to support the Hypothesis 2. And in Section 6.6 some similar but more detailed analyses are conducted to further analyze the impacts of the skipping periods over each components of the trend factor. Instead of analyzing the contribution of the SREV, MOM and LREV factors, in this section I constructed portfolios based on each lag of the trend factor, and analyzed the contribution of all these portfolios over the trend factor.

Firstly, as the same in equation (1), the normalized moving averages $\tilde{A}_{jt,L}$ of stock j at the end of month j, with lag L in 3-, 5-,10-, 20-. 50-, 100-, 200-, 400-, 600-, 800-, and 1000-day length are calculated.

Secondly, instead of using cross-section regression of all 11 lags, only the normalized moving averages $\tilde{A}_{jt,L}$ of stock j at the end of month j, with the same lag L are included in the regression. And since there are in total 11 different lag lengths, 11 individual regressions are conducted over the normalized moving averages of each lag length, where $r_{j,t}$ is the return of stock j in month t, $\beta_{i,t}$ is the coefficient of the normalized MA with leg L_i in month t, and $\beta_{0,t}$ is the intercept in month t.

$$r_{j,t} = \beta_{0,t} + \beta_{\mathrm{i},t}\tilde{A}_{jt-1,L_i} + \varepsilon_{j,t}\ , \qquad j = 1, \dots, n \tag{6}$$

Then similar methods as described in equation (3) and equation (4) are used to construct 11 single lag portfolios. The difference between those single lag portfolios and the trend factor is that the trend factor combines the information of 11 different lag lengths, while each of those single lag portfolios only contains the information of the related lag length. Intuitively, those 11 individual portfolios can be seen as the decomposed trend factor.

The same summary statistics as in Section 6.1 and 6.2 are reported in Table 7, for the trend factor, those 11 single lag portfolios and the related benchmarking SREV, MOM and LREV factors. The portfolios MA3, MA5, MA10 and MA20 are the single lag portfolios with short-term lag lengths, the monthly returns of those portfolios are between 1.44% (MA20) and 1.67% (MA10), and the Sharpe ratios are between 0.52 (MA3 and MA5) and 0.39 (MA20); The portfolios MA50, MA100 and MA200 are the single lag portfolios with mid-term lag lengths, the monthly returns of those portfolios are between 0.28% (MA200) and 1.08% (MA50), and the Sharpe ratios are between 0.08 (MA200) and 0.28 (MA50); The portfolios MA400, MA600, MA800 and MA1000 are the single lag portfolios with long-term lag lengths, the monthly returns of those portfolios are between -0.03% (MA600 and MA800) and 0.09% (MA400), and the Sharpe ratios are between -0.01 (MA600 and MA800) and 0.02 (MA400). It is very obvious that the short-term lag related portfolios have higher returns and the long-term lag related portfolios have lower returns, and within each group, it's also true that with the increasing length of the lag, the monthly return as well as the Sharpe ratio decline (except for MA600 and MA800, which are not statistically significant), which is consistent with the observation that both monthly return and Sharpe ratio rankings are SREV > MOM > LREV.

Table 7. Performances of the single lag portfolios

This table provides the same summary statistics as Table 1, including mean, standard deviation, Sharpe ratio, skewness and excess kurtosis, for the trend factor, as well as the 11 components of the trend factor, which using single lag (instead of the 11 lags) to form the portfolio, and the benchmarking short-term reversal factor (SREV), the momentum factor (MOM) and the long-term reversal factor (LREV). The single lag portfolios are name with MA- and the lag length, for example, portfolio "MA3" is the portfolio formed only based on the 3-day lag and use the same logic can calculations. The t-statistics are in parentheses and significance at 1% level is given by ***, 5% level by **, and 10% level by *.

Factor	Mean (%)	Std. dev (%)	Sharpe ratio	Skewness	Excess kurtosis
Trend	1.69*** (13.41)	4.02	0.42	1.50	19.77

MA3	1.47*** (16.49)	2.77	0.52	2.53	16.66
MA5	1.60*** (16.63)	3.07	0.52	2.09	13.9
MA10	1.67*** (15.42)	3.45	0.48	1.85	14.31
MA20	1.44*** (12.35)	3.72	0.39	1.55	17.03
MA50	1.08*** (9.03)	3.82	0.28	2.42	16.15
MA100	0.64*** (5.16)	3.96	0.16	3.09	22.72
MA200	0.28** (2.46)	3.57	0.08	3.76	31.67
MA400	0.09 (0.75)	3.80	0.02	5.76	75.37
MA600	-0.03 (-0.28)	3.72	-0.01	2.41	21.79
MA800	-0.03 (-0.23)	4.01	-0.01	2.06	25.57
MA1000	0.06 (0.41)	4.46	0.01	3.92	46.31
SREV	0.79*** (7.21)	3.49	0.23	0.99	8.18
MOM	0.79*** (3.26)	7.69	0.10	-4.41	40.42
LREV	0.34*** (3.09)	3.50	0.10	2.93	24.76

Then similar to the Sharpe style regressions in Section 6.5, the return of the trend factor is regressed over the returns of those 11 single lag portfolios, under the whole sample period, recession period and expansion period, and the results are reported in Table 8. Please note that this regression is the same as the regression conducted for Table 6, while doesn't include the constraints of Sharpe style regressions.

During the whole sample period, the results show the returns of the single lag portfolios MA5 (with a coefficient of 0.51) and MA600 (with a coefficient of 0.55) have the highest coefficient, i.e. contribution to the movement of the trend factor's return. The 5-day (one week, in trading days) and the 600-day (three years, in trading days) lags correspond to the short-term and long-term factors, which is in line with the previous results from the Sharpe style regressions in Section 6.5 – Table 5 that LREV has the largest coefficient and SREV has the second largest coefficient. The results in recession periods show the key driver portfolio MA5's coefficient increases to 0.61 and MA600's coefficient declines to 0.30 (not statistically significant), while the results in expansion periods show MA5's coefficient declines to 0.32 and MA600's coefficient increases to 0.58, again these findings are in line with the changes of SREV and LREV's coefficients during different periods from the previous analysis.

However, not all the other single lag portfolios related to the SREV and LREV show the same patterns. For example, MA3 and MA10, the single lag portfolios related to SREV, of which the coefficients decline significantly in recession period and recover in expansion period, show opposite patterns of MA5; and MA400, MA800 and MA1000, the rest single lag portfolios related to LREV, also show opposite patterns of MA600. When compare different results between single lag portfolios (Table 8) and the SREV, MOM and LREV factors (Table 5), it should be pointed out the differences between those two sets of portfolios/factors. According to Kenneth R. French's data library (French, 2018), the SREV factor is formed by a double sorting based on 2 portfolios of size (market equity, ME) and 3 portfolios of priori 1-month return, and the SREV's return is calculated as (Small Low + Big Low)/2 – (Small High + Big High)/2, while the single lag portfolios are built with HZZ's method.

Although not all the results in Table 8 are the same as the previous Sharpe style regressions results, it is still clear to identify the key drivers of the trend factor's return movement are the short-term lag MA5 portfolio and long-term lag MA600 portfolio. And those results merely serve as a starting point, and further analyses are needed to explore the impacts of skipping period would require some further analyses on the relationships, which will be given in the following parts.

Table 8. Regressions of the trend factor on single lag portfolios

The table reports the results of regressing the trend factor's return on the single lag portfolios' returns. The sample period is from January 1931 to December 2014, including 1008 months (observations). The t-statistics are in parentheses and significance at 1% level is given by ***, 5% level by **, and 10% level by *.

	Whole sample	Recession	Expansion
Intercept	0.63***	1.15***	0.61***
	(5.29)	(3.05)	(5.10)
MA3	0.16*	-0.04	0.31***
	(1.76)	(-0.19)	(3.07)
MA5	0.51***	0.61**	0.32***
	(4.61)	(2.18)	(2.66)
MA10	-0.08	-0.19	-0.03
	(-0.84)	(-0.79)	(-0.27)
MA20	0.06	0.07	0.10
	(0.76)	(0.34)	(1.18)
MA50	0.19**	0.32*	0.12
	(2.47)	(1.83)	(1.40)
MA100	-0.20***	-0.16	-0.35***
	(-2.91)	(-1.16)	(-4.03)
MA200	0.03	-0.21	0.25***
	(0.42)	(-1.55)	(3.01)
MA400	-0.45***	-0.28**	-0.42***
	(-7.14)	(-2.03)	(-4.87)
MA600	0.55***	0.30	0.58***
	(7.48)	(1.06)	(7.64)
MA800	-0.32***	-0.01	-0.89***
	(-3.76)	(-0.02)	(-6.55)
MA1000	0.20***	0.15***	0.63***
	(3.02)	(1.03)	(5.77)

Similar to previous analyses on the SREV, MOM and LREV factors, in order to further explore the impacts of the skipping periods on each of those 11 single lag portfolios, I further conduct regressions of the trend factor with skipping periods on those single lag portfolios. Table 9 reports the results of the regressions, in which the Panel A are the regressions on the trend factor with "excluding types" skipping period and Panel B are related to the "inserting types". And Figure 3 visualizes those data included in Table 9, to show the changes of the coefficients in a more convenient way.

The previous Sharpe style regressions on the SREV, MOM and LREV factors in Section 6.5 Table 5 show that with the length of skipping periods becomes longer, the SREV's coefficient will decline and LREV's coefficient will grow larger. However, the results in Table 9 don't show exactly the same patterns. Overall, there isn't a clear decreasing trend of short-term lag portfolios' coefficients. Although the key driver portfolio MA5's coefficients shows some decline trend under the first type ("excluding type") of skipping period, which first increases from 0.51 (without skipping period, as in Table 8) to 0.70 when the trend factor is applied with 1-day skipping period, but declines to 0.44 and 0.21 after applied with 5-day and 10-day skipping periods (as shown in Table 9 – Panel A). While under the "inserting type" skipping periods, the MA5's coefficient doesn't show a clear trend in its changes (as shown in Table 9 – Panel B). As for the other key driver portfolio MA600, the coefficient bounces between 0.4 and 0.6 under both types of skipping periods, without showing any clear trend.
Despite it is difficult to identify a clear pattern on the changes of coefficients with regard to the length of skipping period, the Table 9 and Figure 3 show that the coefficients of MA50 and MA600, the previously identified key driver portfolios, stay at very high level regardless of the lengths of skipping periods. That is to say, no matter what kind of skipping period is applied to the trend factor, its return is most sensitive to the returns of MA50 and MA600 portfolios.

Table 9. Regressions of the trend factor with skipping periods on single lag portfolios

The table reports the results of regressing the trend factor's return (with skipping periods of both types, where S1 is the length of first type, i.e. "excluding type" skipping period which excludes the most recent observations of the formation period, and S2 is the length of second type, i.e. "inserting type" skipping period which doesn't exclude the most recent observations but insert a gap after formation period) on the single lag portfolios' returns. The sample period is from January 1931 to December 2014, including 1008 months (observations). The t-statistics are in parentheses and significance at 1% level is given by ***, 5% level by **, and 10% level by *.

Panel A	Trend	Trend (S1=1)	Trend (S1=5)	Trend (S1=20)
Intercept	0.63***	0.69***	0.76***	0.83***
	(5.29)	(5.05)	(5.41)	(5.89)
MA3	0.16*	-0.33***	0.05	0.04
	(1.76)	(-3.13)	(0.43)	(0.35)
MA5	0.51***	0.70***	0.44***	0.21
	(4.61)	(5.45)	(3.37)	(1.56)
MA10	-0.08	-0.17	-0.52***	-0.09
	(-0.84)	(-1.60)	(-4.74)	(-0.83)
MA20	0.06	0.06	0.08	-0.17*
	(0.76)	(0.66)	(0.92)	(-1.81)
MA50	0.19**	0.25***	0.19**	0.14
	(2.47)	(2.82)	(2.03)	(1.56)
MA100	-0.20***	-0.38***	-0.40***	-0.47***
	(-2.91)	(-4.76)	(-4.89)	(-5.65)
MA200	0.03	0.07	0.16**	0.12
	(0.42)	(0.95)	(2.10)	(1.60)
MA400	-0.45***	-0.43***	-0.46***	-0.51***
	(-7.14)	(-5.93)	(-6.18)	(-6.87)
MA600	0.55***	0.55***	0.48***	0.51***
	(7.48)	(6.47)	(5.48)	(5.77)
MA800	-0.32***	-0.24**	-0.17*	-0.16
	(-3.76)	(-2.45)	(-1.66)	(-1.60)
MA1000	0.20***	0.18**	0.19**	0.29***
	(3.02)	(2.36)	(2.46)	(3.77)

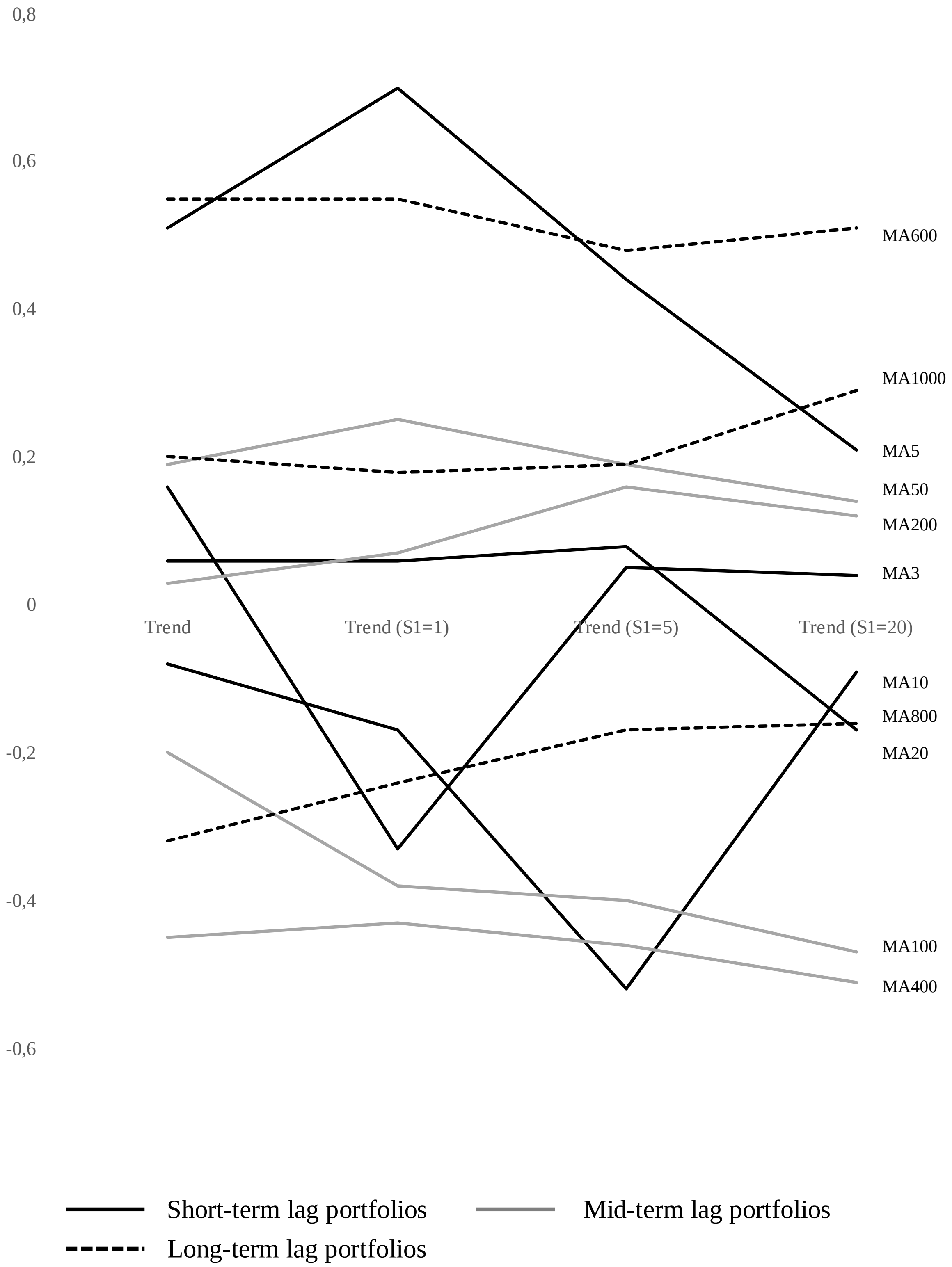
Panel B	Trend	Trend (S2=1)	Trend (S2=5)	Trend (S2=20)
Intercept	0.63***	0.66***	0.78***	0.68***
	(5.29)	(4.86)	(5.42)	(5.20)
MA3	0.16*	-0.30***	0.02	-0.12
	(1.76)	(-2.82)	(0.16)	(-1.21)
MA5	0.51***	0.66***	0.45***	0.61***
	(4.61)	(5.17)	(3.35)	(4.94)
MA10	-0.08	-0.18*	-0.47***	-0.41***
	(-0.84)	(-1.65)	(-4.14)	(-4.05)
MA20	0.06	0.10	0.07	-0.01
	(0.76)	(1.17)	(0.74)	(-0.15)
MA50	0.19**	0.23***	0.09	0.14
	(2.47)	(2.61)	(0.97)	(1.60)
MA100	-0.20***	-0.41***	-0.32***	-0.40***
	(-2.91)	(-5.11)	(-3.76)	(-5.19)
MA200	0.03	0.10	0.28***	0.21***
	(0.42)	(1.40)	(3.62)	(2.94)
MA400	-0.45***	-0.45***	-0.54***	-0.58***
	(-7.14)	(-6.30)	(-7.14)	(-8.45)
MA600	0.55***	0.51***	0.54***	0.44***
	(7.48)	(6.03)	(5.97)	(5.35)
MA800	-0.32***	-0.17*	-0.17*	0.07
	(-3.76)	(1.69)	(-1.66)	(0.76)
MA1000	0.20***	0.13*	0.15*	0.08
	(3.02)	(1.77)	(1.89)	(1.15)

Figure 3. Coefficients of single lag portfolios

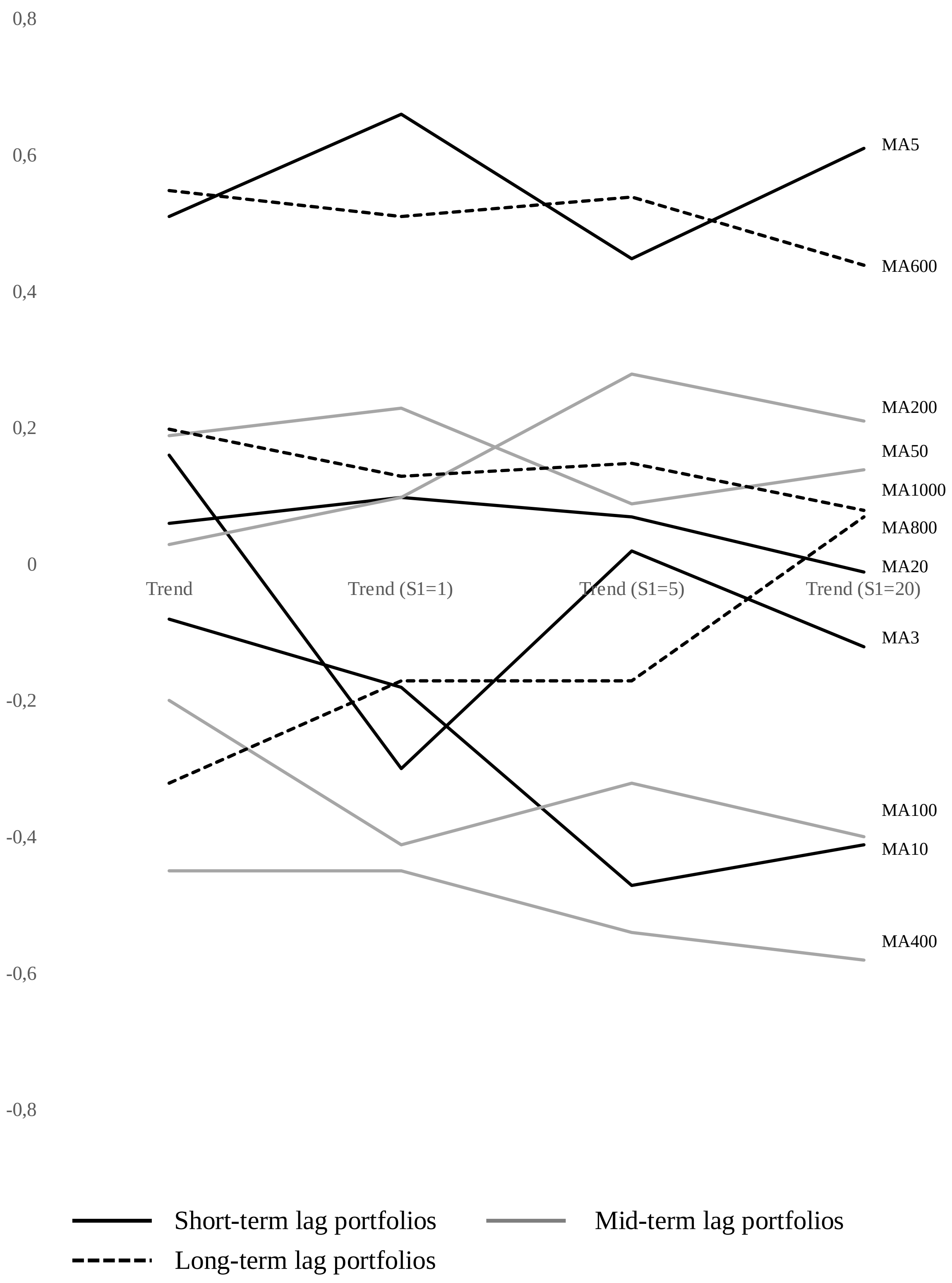
The figures below show the changes of coefficients of the single lag portfolios (from the results of Table 9 – Panel A and Panel B) when different lag lengths of skipping period applied to the trend factor. S1 is the length of first type, i.e. "excluding type" skipping period which excludes the most recent observations of the formation period, and S2 is the length of second type, i.e.

“inserting type” skipping period which doesn’t exclude the most recent observations but insert a gap after formation period.

Coefficients – the first type (“excluding type”) of skipping period:



Coefficients – the second type ("inserting type") of skipping period:



5.8. Contributed Returns of the Single Lag Portfolios

Since the analyses on the 11 single lag portfolios in previous section shows somewhat different results compared to the SREV, MOM and LREV factors. It would be interesting to see the contributed returns of those single lag portfolios. Similar to Section 6.6, the contributed return of each of those portfolios is calculated by multiplying its coefficient from Table 8 (whole sample period) with its average monthly return from Table 7, and the results are reported in Table 10.

Table 10 shows that out of the 1.69% monthly average return of the trend factor, MA5 portfolio accounts for nearly half of the return (0.82%), and MA3 portfolio accounts for the second largest part of the return (0.24%), and overall most of the trend factor's return is contributed by short-term lag portfolios, while the long-term lag portfolios accounts for the least (MA400 accounts for -0.04%, MA600 accounts for -0.02%, MA800 accounts for -0.01% and MA400 accounts for 0.01%). This finding is consistent with the analyses of the SREV, MOM and LREV factors' contributed returns (in Section 6.6) that the SREV factor has the highest contributed return and the LREV factor has the lowest. In addition, when looking at the key driver portfolios to the trend factor's return movement identified in the regressions (in Section 6.7), MA5 also accounts for large part of the trend factor's return (0.82%), while MA600 accounts for a very limited share (0.01%) of the trend factor's return.

The previous analysis on decomposition of the trend factor by SREV, MOM and LREV factors proves the skipping period affects the trend factor mainly through the SREV factor. And the findings from the regressions on the 11 single portfolios (Table 8) and the contributed returns (Table 10) gives us a more detailed view on how the skipping periods affect the trend factor through the 11 related single lag portfolios. The MA5 portfolio is identified as the main driver for the trend factor's performance, as it not only has a relatively very high coefficient, but also account for 0.82%, the largest part of the trend factor's return without any skipping periods. With a 5-day skipping period of the first type, 0.82% out of the 1.69% return of trend factor will be removed. While the results show that the trend factor's return is also sensitive to the long-term lag portfolio MA600, the low return of the MA600 portfolio makes it hard to create significant movements on the trend factor's return. Thus the skipping period affects the trend factor's performance mainly by affecting the short-term related lags, especially the 5-day lag. Again, those results support the Hypothesis 2.

Table 10. Contributed returns of the single lag portfolios

The table reports the contributed monthly average return of the 11 single lag portfolios to the overall monthly average return (1.69%) of the trend factor. The contributed return of each portfolio is multiplied by its coefficient (whole sample) from the regressions, and the average monthly return of the factor. The t-statistics of the coefficient and the average monthly return are in parentheses and significance at 1% level is given by ***, 5% level by **, and 10% level by *.

Factor	Coefficient (whole sample)	Average monthly return (%)	Contributed return (%)
MA3	0.16*	1.47***	0.24
	(1.76)	(16.49)	
MA5	0.51***	1.60***	0.82
	(4.61)	(16.63)	
MA10	-0.08	1.67***	-0.13
	(-0.84)	(15.42)	
MA20	0.06	1.44***	0.09
	(0.76)	(12.35)	
MA50	0.19**	1.08***	0.21
	(2.47)	(9.03)	
MA100	-0.20***	0.64***	-0.13
	(-2.91)	(5.16)	
MA200	0.03	0.28**	0.08
	(0.42)	(2.46)	
MA400	-0.45***	0.09	-0.04
	(-7.14)	(0.75)	
MA600	0.55***	-0.03	-0.02
	(7.48)	(-0.28)	
MA800	-0.32***	-0.03	0.01
	(-3.76)	(-0.23)	
MA1000	0.20***	0.06	0.01
	(3.02)	(0.41)	

6. Conclusion

HZZ's trend factor provides a combined factor from the short-term reversal factor, the momentum factor, and the long-term reversal factor through cross-section regressions in order to gain abnormal returns. This study is based on HZZ's approach, and further examines the performance of the trend factor under the setting of skipping period. The skipping period is widely used by related studies in order to mitigate the bid-ask spread bias and avoid the opposite effects from shorter-term factors. The skipping period also provides a practical setup which considers the real-life trades execution issues.

The trend factor is examined under two types of skipping periods, the first type ("excluding type"), and the second type ("inserting type"), which are widely used by studies on short-term reversal effect, mid-term momentum effect, and long-term reversal effect. And there are three lengths for each type of the skipping period tested in this study: 1-day, 5-day and 20-day length, which correspond to the 1-day, 1-week and 1-month period which are widely used by many other studies as the lengths for skipping periods.

This study aims to test two hypotheses: one is that the skipping period will reduce the return of the trend factor, and the other is that such impacts are mainly through the SREV factor.

Hypothesis 1 is supported by the results that return of the trend factor declines significantly after the application of skipping period. Under both the "excluding type" and the "inserting type" of skipping periods, the monthly average return of trend factor drops from 1.69% by more than 0.50% when only the 1-day skipping periods are applied, and after applying the 5-day and 20-day skipping periods the return of the trend factor becomes lower than that of the SREV and MOM factor. The alpha of the trend factor also declines sharply, and after applying the skipping period it becomes lower than the MOM factor's alpha (in terms of both CAPM alpha and FF's three factor alpha).

Then, the Hypothesis 2 is supported by decomposing the trend factor and analyzing the contributed returns of its components. The decomposition of trend factor by SREV, MOM and LREV factors suggests the declines of trend factor's performance is mainly driven by the SREV factor, as it accounts for the most parts of trend factor's returns (0.24% out of 1.69%), and with the length of skipping period increases, the coefficient between the trend factor's return and the SREV's return declines. Together those two mechanisms lead the return of the trend factor drop significantly when the skipping period becomes longer. In addition, in the study trend factor is further decomposed by its lags into 11 single lag portfolios, and the short-term MA5 portfolio is identified as the main driver of trend factor's return, as it not only has a very high coefficient with the trend factor's return (0.21 to 0.51, depends on the length of skipping period), but also accounts for the most part of trend factors' return (0.82% out of 1.69%). The results indicate that the skipping period impacts trend factor's performance through the SREV factor.

To summarize, the study finds that with the skipping period, the performance of the trend factor declines largely and its superiority over other factors disappears. The study also shows that the impacts of skipping period over the trend factor's is mainly due to the short-term reversal factor, especially the 5-day lag of the trend factor.

This study contributes to the previous research on the trend factor mainly in two ways. Firstly, the study examines HZZ's trend factor under the context of skipping period, the application of skipping period not only serves as a consistent method as most peer studies, but also provides evidence on the trend factor's performance in a real-world setting. Secondly, the analyses on decomposition of the trend factor also provides some details on the sources of trend factor's abnormal returns, which suggest the high returns of the trend factor is mainly from its short-term lags, which could be further explored and leveraged by future studies.

7. References

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8. Appendices

Appendix A. The original table of summary statistics from HZZ's study

Factor	Mean (%)	Std. dev (%)	Sharpe ratio	Skewness	Excess kurtosis
Trend-HZZ	1.63*** (15.0)	3.45	0.47	1.47	11.3
SREV	0.79*** (7.21)	3.49	0.23	0.99	8.22
MOM	0.79*** (3.29)	7.69	0.10	-4.43	40.7
LREV	0.34*** (3.09)	3.50	0.10	2.93	24.8
Market	0.62*** (3.69)	5.40	0.12	0.27	8.03
SMB	0.27*** (2.63)	3.24	0.08	2.04	19.9
HML	0.41*** (3.64)	3.58	0.11	2.15	18.9